Logarithm - Important Formulas

1. **Logarithm:**

   If \( a \) is a positive real number, other than 1 and \( a^m = x \), then we write: \( m = \log_a x \) and we say that the value of \( \log x \) to the base \( a \) is \( m \).

   **Examples:**

   (i). \( 10^3 \ 1000 \quad \Rightarrow \quad \log_{10} 1000 = 3. \)

   (ii). \( 3^4 = 81 \quad \Rightarrow \quad \log_3 81 = 4. \)

   (iii). \( 2^{-3} = \frac{1}{8} \quad \Rightarrow \quad \log_2 \frac{1}{8} = -3. \)

   (iv). \( (.1)^2 = .01 \quad \Rightarrow \quad \log_{(.1)} .01 = 2. \)

2. **Properties of Logarithms:**

   1. \( \log_a (xy) = \log_a x + \log_a y \)

   2. \( \log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y \)

   3. \( \log_x x = 1 \)

   4. \( \log_a 1 = 0 \)

   5. \( \log_a (x^n) = n(\log_a x) \)

   6. \( \log_a x = \frac{1}{\log_x a} \)

   7. \( \log_a x = \frac{\log_b x}{\log_b a} \)

3. **Common Logarithms:**

   Logarithms to the base 10 are known as common logarithms.

   4. The logarithm of a number contains two parts, namely 'characteristic' and 'mantissa'.

   **Characteristic:** The internal part of the logarithm of a number is called its **characteristic.**

   Case I: When the number is greater than 1.
In this case, the characteristic is one less than the number of digits in the left of the decimal point in the given number.

Case II: When the number is less than 1.

In this case, the characteristic is one more than the number of zeros between the decimal point and the first significant digit of the number and it is negative.

Instead of -1, -2 etc. we write 1 (one bar), 2 (two bar), etc.

Examples:

<table>
<thead>
<tr>
<th>Number</th>
<th>Characteristic</th>
<th>Number</th>
<th>Characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>654.24</td>
<td>2</td>
<td>0.6453</td>
<td>1</td>
</tr>
<tr>
<td>26.649</td>
<td>1</td>
<td>0.06134</td>
<td>2</td>
</tr>
<tr>
<td>8.3547</td>
<td>0</td>
<td>0.00123</td>
<td>3</td>
</tr>
</tbody>
</table>

**Mantissa:**

The decimal part of the logarithm of a number is known is its **mantissa**. For mantissa, we look through log table.