1. Ratio:

The ratio of two quantities \( a \) and \( b \) in the same units, is the fraction \( \frac{a}{b} \) and we write it as \( a : b \).

In the ratio \( a : b \), we call \( a \) as the first term or antecedent and \( b \), the second term or consequent.

Eg. The ratio 5 : 9 represents \( \frac{5}{9} \) with antecedent = 5, consequent = 9.

Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Eg. 4 : 5 = 8 : 10 = 12 : 15. Also, 4 : 6 = 2 : 3.

2. Proportion:

The equality of two ratios is called proportion.

If \( a : b = c : d \), we write \( a : b :: c : d \) and we say that \( a, b, c, d \) are in proportion.

Here \( a \) and \( d \) are called extremes, while \( b \) and \( c \) are called mean terms.

Product of means = Product of extremes.

Thus, \( a : b :: c : d \Leftrightarrow (b \times c) = (a \times d) \).

3. Fourth Proportional:

If \( a : b = c : d \), then \( d \) is called the fourth proportional to \( a, b, c \).

Third Proportional:

\( a : b = c : d \), then \( c \) is called the third proportion to \( a \) and \( b \).

Mean Proportional:

Mean proportional between \( a \) and \( b \) is \( \sqrt{ab} \).

4. Comparison of Ratios:

We say that \( (a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d} \).
5. **Compounded Ratio:**

6. The compounded ratio of the ratios: \((a : b), (c : d), (e : f)\) is \((ace : bdf)\).

7. **Duplicate Ratios:**

Duplicate ratio of \((a : b)\) is \((a^2 : b^2)\).

Sub-duplicate ratio of \((a : b)\) is \((a : b)\).

Triplicate ratio of \((a : b)\) is \((a^3 : b^3)\).

Sub-triplicate ratio of \((a : b)\) is \((a^{1/3} : b^{1/3})\).

\[
\frac{a}{b} \quad \frac{c}{d} \quad \frac{a + b}{a - b} \quad \frac{c + d}{c - d} \]

If \(\frac{a}{b} = \frac{c}{d}\), then \(\frac{a + b}{c + d} = \frac{a - b}{c - d}\). [componendo and dividendo]

8. **Variations:**

We say that \(x\) is directly proportional to \(y\), if \(x = ky\) for some constant \(k\) and we write, \(x \propto y\).

We say that \(x\) is inversely proportional to \(y\), if \(xy = k\) for some constant \(k\) and we write, \(x \propto \frac{1}{y}\).