1. (a) Test the convergence of \[ \sum \frac{1}{n^2 + 1} \] 
(b) Find the sum to infinity of the series \[ \frac{15}{16} + \frac{15.21}{16.24} + \frac{15.21.27}{16.24.32} + \ldots \]

(c) Sum the series \[ \frac{1}{2} + \frac{1+3}{2} + \frac{1+3+3^2}{2} + \frac{1+3+3^2+3^3}{2} + \ldots \]

Time: Three hours
Answer any FIVE questions.
Maximum: 100 marks

(5 \times 20 = 100)
2. (a) State and prove Rabe's test.
(b) Discuss the convergence of the series.
(c) Test the convergence of the series.

\[ \sum_{n=1}^{\infty} \frac{1}{n^2+1} \]

3. (a) Find the evolutes of the ellipse.
(b) Find the radius of curvature for the curve.

\[ y^2 = x^3 + 8 \text{ at } (-2,0) \]

4. (a) By change the order of integration.
(b) Evaluate the integral.

\[ \int_{0}^{1} \left( \int_{0}^{y} (x+y+z) \, dx \right) \, dy \]

5. (a) Find the radius of curvature for the curve.
(b) Find the positive real root of the equation.

\[ x^3 - 3x + 1 = 0 \text{ by Horner's method.} \]

\[ x^2/a^2 + y^2/b^2 = 1 \]

6. (a) 15
(b) 15.21
(c) 16.24
(d) 15.21.27

7. (a) \[ \sum_{n=1}^{\infty} \frac{1}{n^2+1} \]
(b) \[ \sum_{n=1}^{\infty} \frac{1}{n^2+1} \]
(c) \[ \sum_{n=1}^{\infty} \frac{1}{n^2+1} \]
(d) \[ \sum_{n=1}^{\infty} \frac{1}{n^2+1} \]
(e) \[ \sum_{n=1}^{\infty} \frac{1}{n^2+1} \]
(f) \[ \sum_{n=1}^{\infty} \frac{1}{n^2+1} \]
(g) \[ \sum_{n=1}^{\infty} \frac{1}{n^2+1} \]
(h) \[ \sum_{n=1}^{\infty} \frac{1}{n^2+1} \]
(i) \[ \sum_{n=1}^{\infty} \frac{1}{n^2+1} \]
(j) \[ \sum_{n=1}^{\infty} \frac{1}{n^2+1} \]
polar co-ordinates.

Evaluate \( \iiint_V x p(x^2 + y^2 + z^2) \, dx \, dy \, dz \) where \( V \) is the volume of the sphere.

Prove that \( \int_0^1 \int_0^1 \int_0^1 3x + 3y + 3z \, dx \, dy \, dz = 3 \).

Find the area included between the curves \( y = x^2 \) and \( y = x \).

Simpson's one-third rule with \( n = 10 \).

Trapezoidal rule.

Find a real root of the equation \( 2x^3 - 3x - 6 = 0 \).

Evaluate \( \int_0^1 \int_0^1 \int_0^1 x \, dx \, dy \, dz \).

Find the relation between Beta and Gamma.
(b) Verify Euler's theorem from the function

\[ u = \cos^{-1} \left( \frac{x + y}{\sqrt{x + y}} \right) \]

(i) \( ab = c^2 \) where \( c \) is a constant.
(ii) \( a + b = c \)

Find the envelope of the family of straight lines \( x + y / a = 1 \) where \( a \) and \( b \) are connected by the relation \( x^2 + y^2 + 2 = 1 \) and \( x + y = 1 \) given.

\[ \int \left( 1 + x + y \right) \left( x - 1 \right) \left( 2x + 3 \right) \, dx \]

\[ x^2 = 4y \]

\[ x = \frac{4y}{a} \]

(iii) \( x^2 \) is a constant.
I. (a) Prove that
\[ \tan \left( \log \frac{a - ib}{a + ib} \right) = \frac{2ab}{a^2 - b^2}. \]
(b) Prove that
\[ \frac{\cos \theta \sin^3 \theta}{2} = \frac{1}{2} \sin 6\theta + 2 \sin 4\theta - 6 \sin 2\theta. \]
2.
(a) \( \tan \left( \frac{\log a - \log b}{2} \right) = \frac{a^2 - b^2}{2 \log b} \) show that

(b) \[ \sum_{n=1}^{\infty} \frac{x^2}{\sinh^2 x} = 1 \]

2. \[ \cos^3 \theta \sin^5 \theta = \frac{1}{2!} \sin^6 \theta + 2 \sin^4 \theta - 6 \sin^2 \theta \]

3. \[ F = (6x + z^2) \hat{i} + (3x^2 - z) \hat{j} + (3x^2 - y) \hat{k} \]

(a) Show that the field \( F \) is irrotational and find its scalar potential.

4.
(a) Evaluate \[ \int \text{div}(\nabla \times \mathbf{A}) \cdot d\mathbf{S} \]

\[ \mathbf{F} = \nabla \times \mathbf{A} + (\nabla \cdot \mathbf{A}) \mathbf{A} \]

(b) Verify Green's theorem in the \( x \)-\( y \) plane for the surface of the sphere \( x^2 + y^2 + z^2 = 1 \) which lies in the first octant.

3. \[ \int \text{div}(\nabla \times \mathbf{A}) \cdot d\mathbf{S} \]

\[ \mathbf{A} = (\mathbf{V} \times \mathbf{B}) \mathbf{V} - (\mathbf{V} \times \mathbf{B}) \]

4. \[ \int \text{div}(\nabla \times \mathbf{A}) \cdot d\mathbf{S} \]

\[ \mathbf{F} = (6x + z^2) \hat{i} + (3x^2 - z) \hat{j} + (3x^2 - y) \hat{k} \]

(a) Evaluate \[ \int \text{div}(\nabla \times \mathbf{A}) \cdot d\mathbf{S} \]

(b) Verify Green's theorem in the \( x \)-\( y \) plane for the surface of the sphere \( x^2 + y^2 + z^2 = 1 \) which lies in the first octant.

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\[ \mathbf{F} = (6x + z^2) \hat{i} + (3x^2 - z) \hat{j} + (3x^2 - y) \hat{k} \]

(a) Evaluate \[ \int \text{div}(\nabla \times \mathbf{A}) \cdot d\mathbf{S} \]

(b) Verify Green's theorem in the \( x \)-\( y \) plane for the surface of the sphere \( x^2 + y^2 + z^2 = 1 \) which lies in the first octant.
5. (a) Express $f(x) = (\pi - x)^2$ as a Fourier series of periodicity $2\pi$ in $0 < x < 2\pi$ and hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. 

(b) Find Fourier series of periodicity 2 for $f(x) = \begin{cases} x+2 & -1 \leq x \leq 1 \\ \text{sum of } 1, -1, 1, -1, 1, \ldots \end{cases}$. 

6. (a) In any conic, prove that the sum of the reciprocals of two perpendicular focal chords is constant. 

(b) Show that the equations $\frac{1}{r} = 1 + \varepsilon \cos \theta$ represent the same conic. 

7. (a) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ are coplanar and find the equation of the plane containing them.
Find the locus of the point of intersection of the three mutually perpendicular tangent planes at the center of the conicoid $ax^2 + by^2 + cz^2 = 1$.

Find the equation of the right circular cone whose vertex is at the origin, whose axis is the line $\frac{x}{z} = \frac{y}{z} = \frac{1}{x}$, and which has a vertical angle of $60^\circ$.

Find the equation to the right circular cone whose axis is $2x + 3y + 6z = 0$ and whose vertex is at the origin.

Center and radius of the sphere having circle $2x - y + 6z = 0$ as great circle.
1. (a) State and prove multiplication theorem on expectation.
   (b) A random variable $X$ has the following probability density function:
   $f(x) = Ax^2$ for $0 \leq x \leq 1$.

Answer any FIVE questions.

Maximum: 100 marks

Time: Three hours
moment of Binomial distribution

3. Obtain the recurrence relation for the
   $1 \leq x \leq 1$.
   \[ \frac{x}{1+x} = (x)f \]

2. Obtain the MGF of $f$.

(a) $MGF$ of $f$.

(b) State and prove addition theorem of MGF.

(c) State and prove Chebychev's Inequality.

3. (a) Obtain the minimum

(b) Find the minimum

(c) $\frac{x}{1}< (X > X)d$.

(d) $(0 < x < 0)d$

(e) $(x > X)d$ and $(6 > X)d$

(f) $(6 > X)d$

(g) Find $X$

(h) Find $X$

(i) $(X > X)d$

(j) $0 > X > X$

(k) $p(X) = 0$,

(l) $X = 1, 2, 3, 4, 5, 6, 7$

A probability function

A random variable $X$ has the following

from $0$ to $\infty$. Find mean and variance.

A probability function has a range

\[ \frac{x}{1} > \frac{x}{1} \]

\[ (0.3 > X)d \]

\[ (0.0 > X > 0.0)d \]

\[ (0.2) \]

\[ (0.9) \]

\[ (0.8) \]

\[ (0.9) \]
5. (a) Fit a straight line to the following data:
\[ \begin{array}{c|c}
X & Y \\
\hline
1 & 3.6 \\
2 & 3.6 \\
3 & 6.8 \\
4 & 0.1 \\
5 & 4.3 \\
6 & 1.8 \\
7 & 2.7 \\
8 & 6.3 \\
\end{array} \]

(b) Find the correlation coefficient.

6. (a) A die is thrown 132 times with the following results:
\[ \begin{array}{c|c}
X & Y \\
\hline
1 & 125 \\
2 & 137 \\
3 & 156 \\
4 & 112 \\
5 & 107 \\
6 & 136 \\
\end{array} \]

(b) Obtain the MGF of Normal Distribution.

(c) Obtain the MGF of Poisson distribution.

7. (a) Find the first four moments of Poisson distribution
\[ f(x) = \frac{2e^{-x}}{1+x} \text{ for } x \geq 1. \]
(b) Find the two lines of regression for the