# H.N.B. Garhwal University Srianagar Garhwal

# **Department of Mathematics**

# M.A/M.Sc. Mathematics Course Structure

(Applicable to the Candidates admitted from the Academic year 2015-16 onwards)

# Semester - I

## **Core Course**

Paper I –	Discrete Structures	SOS/Math/ C001
Paper II –	Abstract Algebra	SOS/Math/C002
Paper III –	Mechanics	SOS/Math/C003
Paper IV –	Complex – Analysis	SOS/Math/C004
Paper V –	Operations Research	SOS/Math/C005
Paper VI-	Practical	SOS/Math/C006

# **Semester II**

#### **Core Course**

Paper VII	Abstract Algebra II	SOS/Math/C007
Paper VIII	Fluid Dynamics (Remove V <sup>th</sup> Unit)	SOS/Math/C008
Paper IX	Operations Research II	SOS/Math/C009
Paper X	Graph Theory	SOS/Math/C0010
Paper XI	Topology I	SOS/Math/C0011
Paper XII	Practical	SOS/Math/C0012

## **Semester III**

## **Core Course**

Paper XIII	Topology II	SOS/Math/C0010
Paper XIV	Measure Theory I	SOS/Math/C0014
Paper XV	Differential Geometry	SOS/Math/C0015

# **Elective Course**

Paper XVI	Fluid Mechanics	SOS/Math/E001
Paper XVII	Calculus of Variation	SOS/Math/E002
Danar VVIII	Computer fundamentals	and data etructures SOS/Math/F

Paper XVIII Computer fundamentals and data structures SOS/Math/E003

Paper XIX Algebraic Coding theory SOS/Math/E004

# **Self-Study Course-**

Paper XX From the other Departments (Any one of the following) SOS/Math/E005

- (a) Mathematical Techniques
- (b) Tensor Analysis and Special Theory of Relativity
- (c) Financial Mathematics

Core Course Paper XXI Viva-voce SOS/Math/C0016

## **Semester IV:**

## **Core Course**

Paper XXII Measure Theory II SOS/Math/C0017

Paper XXIII Integral Equations and boundary value problems SOS/Math/C0018

Paper XXIV F	Functional Analysis	SOS/Math/C0019
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**Elective Course** 

Paper XXV Biomechanics SOS/Math/E006
Paper XXVI Fuzzy set theory SOS/Math/E007
Paper XXVII Mathematical modeling SOS/Math/E008

**Core Course** 

Paper XXIX Viva-voce SOS/Math/C0020

#### Note:

- 1- In Semester I, all paper are compulsory.
- 2- In Semester II all paper are compulsory.
- 3- In Semester III, Papers XIII, XIV, XV & XXI are compulsory and choose any two papers out of the Elective Papers XVI, XVII, XVIII & XIX.
- 4- In Semester V, Papers XXII, XXIII, XXIV & XXIX are compulsory and choose an two out of the Elective Papers XXV, XXVI, XXVII & XXVIII.
- 5- Each paper carries 100 Marks, which includes two sessional tests (each of 20 Marks) and a main Examination of 60 Marks.

#### SEMESTER - I

## PAPER- I DESCRETE STRUCTURES

# SOS/Math/C001

- I. Recurrence relations, linear homogenous recurrence relations, solutions of recurrence relations.
- II. Partially ordered sets, different type of lattices, sub lattices, direct product, Ideal lattice, modular and distributive lattices.

- III. Boolean algebra, Ideals in Boolean algebra, Boolean rings, Boolean functions, Karaugh maps, application of Boolean algebra to switching theory.
- IV. Graph, direct graph, undirected graph, relations and graphs, path and circuits, Eulerian and Hamiltonian graphs, planner graphs, connected graphs.

#### TEXT BOOKS

1. Discrete Mathematics: Khanna & Bhambri

Element of Discrete Mathematics: C. I Liu
 Discrete Mathematics: G.S. Rao
 Lattice Thoery: V.K. Khanna
 Discrete Mathematics: R. Johnsoubaugh

#### **SEMESTER-I**

# PAPER-II ABSTRACT ALGEBRA- I SOS/ Math/C002

- I. Class equation and conjugacy classes, Cauchy's theorem for finite abelian and non-abelian groups, Sylow's theorems.
- II. The normal series and composition series, Jordon-Holder theorem, Solvable groups, External and internal direct products.
- III. Ideals, Principal ideals, Quotient rings, Field of quotients, embedding of rings, fundamental theorem on homomorphism and isomorphism.
- IV. Prime and maximal ideals, Ring of polynomials, Factorization of polynomials over a field, Factorization theorem in integral domain.

#### **TEXT BOOKS**

Contemporary Abstract Algebra : Josheph A Gallian
 A First course in Abstract Algebra : John. B. Fraleigh

3. Modern Algebra: Surject Singh and Quazi Jameerudin

4. Topics in Algebra: I. N. Herstein

## SEMESTER -I

# PAPER -III MECHANICS SOS/ Math/C003

- I. Conservation of linear and angular momentum under finite and impulsive forces, Conservation of energy.
- II. Generalized coordinates, Lagrange's equations of motion, Small oscillations.
- III. Hamiltonia's canonical equations, Hamilton's principle and principle of least action.
- IV. Euler's equations of motion, Kinetic energy Eulerian angles, Instantaneous axis of rotation.

#### **TEXT BOOKS**

Dynamics- Part II: A.S. Ramsay
 Classic mechanics: H. Goldstein

3. Analytical Mechanics : L.N. Hand and J. Finch4. Classical Mechanics : N.C. Rana and P.S. Juag

5. Dynamics of Rigid Body: Ray and Sharma

6. Dynamics of Rigid body: S.L. Loney

#### **SEMESTER-I**

#### PAPER-IV COMPLEX ANALYSIS SOS/ Math/C004

- I. Complex Integration, Expansion of an angle function as power series, Taylors and Laurents series, Residue and Poles, Singularities, Classification of Isolated singularities, Cauchy Residue Theorem.
- II. Application of Residue Theorem in evaluation of Improper Real Integers and evaluation of sum.
- III. Conformal Mapping Properties, Mobius Transformation, Elementary examples.
- IV. Maximum Modulus Theorem, Mittag-Leffer Theorem, Weirstrases factorization Theorem, Jensen's formula, Poisson Jensen formula, Hadmard, three circle Theorem, Analytic Continuation.

# **TEXT BOOKS**

Real and Complex Analysis: W. Rudin
 Complex Analysis: J.B. Convey
 Complex Analysis: B. Chaudhary
 Complex Analysis: E.C. Tichmarch
 Foundation of Complex Analysis: S. Ponnusomy

#### SEMESTER -I

#### PAPER -V OPERATIONS RESEARCH -I SOS/ Math/C005

- I. Operation research: an introduction, Methodology of O.R. Features of O.R.
   Problems, Applications of O.R. Models Opportunities and shortcomings of O.R.
   Approach.
- II. Dual simples method, Revised simplex method, Sensitivity analysis.
- III. Assignment and transportation problems.
- IV. Theory of games, Integer Linear Programming.

#### **TEXT BOOKS**

1. Operations Research: Kanti Swarup, P.K. Gupta

Operations Research: Theory and Applications : J.K. Sharma
 Operations Research : H.A. Taha

4. Operations Research:

R. Bronson

#### **SEMESTER-I**

#### PAPER-VI PRACTICAL

SOS/ Math/C006

Based on Paper-I to Paper-V

#### **SEMESTER-II**

#### PAPER-VII ABSTRACT ALGEBRA-II SOS/ Math/C007

- I. Introduction to fields extensions, Finite fields, Algebraic extensions, Simple field Extension, Roots of Polynomials.
- II. Splitting Fields, Separable and inseparable extensions, Perfect field.
- III. Automorphisms of fields, Fixed fields, Galois's Theory, Illustrations of Galois's theory.
- IV. Radical extension and solvability, Constructible numbers. The impossibility of certain constructions.

#### **TEXT BOOKS**

Contemporary Abstract algebra : Josheph A. Gallian
 A first course in Abstract Algebra : John B. Fraleigh

3. Modern Algebra: Surjeet Singh and Quazi Zameerudin

4. Topics in Algebra: I. N. Herstein

#### **SEMESTER-II**

#### PAPER-VIII FLUID DYNAMICS SOS/ Math/C008

- I. Kinematics of fluids, Lagrangian and Eulerian methods, Local and individual time rates of change, Equation of continuity, Boundry surface.
- II. Equation of motion of inviscid fluids, Euler's equation of motion, Bernouille's equation, Lanrage's equation, Conservative field of force, Cauchy's Integral, Helmholt's equation.
- III. Impulsive motion, of a fluid, Energy equation of inviscid fluid, General theory of irrotational motion; connectivity, Flow and circulation, Kelvin's circulation theorem, Stoke's theorem, Permanence of irrotational motions, Green's theorem, Kinetic energy of finite and infinite liquid, Kelvin's minimum energy theorem, Mean value of the velocity potential over a spherical surface.
- IV. Motion in two dimention; Stream function, Complex potential, Source, Sink, Doublet, Complex potential and images with respect to straight line and circle, Milne-Circle theorem, Blausius theorem.

#### **TEXT BOOKS**

Foundation to Fluid Mechanics : S.W. Yuan
 Text book of Fluid Dynamics : F. Chorltron
 Theoretical Hydro-Dynamics : Bansi Lal

4. A text book of Fluid – Dynamics: M. Ray & Sharma

#### **SEMESTER-II**

#### PAPER-IX OPERATIONS RESEARCH-II

SOS/ Math/C009

- I. Inventory control, Deterministic Economic order quality models.
- II. Queuing theory, Symbols and notations, Classification of queue, M/M/I queuing models.
- III. Markow chain, Project Scheduling by PERT/CPM.
- IV. Dynamic programming, non-linear programming, Kuhn-Tueker conditions, Wolfe's modified simplex method.

## **TEXT BOOKS**

1. Operations Research: Kanti Swarup, P.K. Gupta

Operations Research, Theory and Applications : J.K. Sharma
 Operations Research : H.A. Taha
 Operations Research : R. Bronson

### **SEMESTER-II**

# PAPER-X GRAPH THEORY SOS/ Math/C0010

- I. Trees and fundamental circuit, Distance and centers, Binary Trees, Binary search, Spanning trees, Spanning trees, Algorithms, Primes and Kruskals, Dijkststra Algorithm, Fundamental circuits, Spanning trees in a weighted graphs and dual graphs.
- II. Matrix Representation of Graphs.
- III. Chromatic number and chromatic polynomials, Matching covering, Chromatic partitioning.
- IV. Directed graphs, Digraph and Binary relations, Euler's digraph, Directed path & connectedness, Acyclic digraph.

#### **TEXT BOOKS**

1. Basic Graph Theory: Parthswarthy

2. Graph Thoery: N. Deo

3. Graph Theory and Application : C. Vashudev

4. Graph Theory: Harry

#### **SEMESTER-II**

# PAPER-XI TOPOLOGY-I SOS/ Math/C0011

- I. Metric space: open sets, closed sets, closure, interior, exterior, dense and nondense sets, sequence and subsequence in metric space, complete metric spaces, Cantor's intersection theorem, Baire's category theorem.
- II. Definition and example of topological spaces, closed sets closure dense subsets, neighborhood, interior and boundary, accumulation points and derived sets, base and sub bases, subspace and relative topology, Kuratowsli closer operator and neighborhood system.
- III. Continuity and homeomorphism.
- IV. Connectedness: connected and disconnected sets, local connectedness, component and path components, continuity and connectedness, totally disconnected space.

#### **TEXT BOOKS**

1. Topology: A First Course: James R. Munkres

2. General Topology: J. L. Kelly

3. Topology and Modern Analysis: G.F. Simmons

4. General Topology : Seymour Lipschutz

# **SEMESTER-II**

PAPER-XII PRACTICAL SOS/ Math/C0012

Based on Paper -VII to Paper- XI

#### SEMESTER-III

# PAPER-XIII TOPOLOGY II SOS/ Math/C0013

- I. Compact spaces, sequentially compact spaces, local compactness, continuity and compactness.
- II. First and second countable spaces, separabilty and Lindlof of property.
- III.  $T_1$  spaces, Hausdorff spaces, regular spaces, normal space, completely normal spaces.
- IV. Product spaces, Nets and filters, Urysohn's lemma, Tietze extension theorem, Tychonoff's product theorem.

#### **TEXT BOOKS**

1. Topology: A First Course : James R. Munkres

2. General Topology: J.L. Kelly

3. Topology and Modern Analysis: G.F. Simmons

4. General Topology: Seymour Lipschutz

#### **SEMESTER-III**

# PAPER –XIV MEASURE AND INTEGRATION-I SOS/ Math/C0014

- I. Denumerable sets, Uncountable sets, Cardinal numbers.
- II. Lebesgue measure, Measurable sets, Borel sets, Cantor's ternary sets and their properties.
- III. Measurable functions, set of measure zero, The structure of measurable functions.
- IV. Lebesgue Integrals and their properties, Lebesgue integrals for unbounded functions, General Lebesgue integrals.

#### TEXT BOOKS

Real Analysis : H.L. Royden
 An Introduction to Measure and Integration : Inder K. Rana

3. Lebesgue Measure and Integration : P.K. Jain and V.P. Gupta

4. Measure Theory and Integration : G. De. Barra

#### SEMESTER III

# PAPER- XV DIFFERENTIAL GEOMETRY SOS/ Math/

- I. Curves in space; Arc length, Order of contact, Tangent, Normal, Binormal, Osculating, Plane, Serrent-Frenet formulae, Curvature and torsion. Osculating circle and osculating sphere, Helix, Bertand curves.
- II. Behaviour of a curve in the neighbourhood of a point. Concept of a surface, Envelope and developable surface, Parametric curves, Family of the surfaces, Edge of regression, Ruled surfaces, Central points.
- III. Fundamental forms and curvature of surfaces: First fundamental form. Second fundamental form of the surfaces of revolution, Weingarten's equation, Direction coefficients, Family of curves.
- IV. Local non-intrinsic properties of a surface Normal curvature, Principal directions, Principal curvatures, Minimal surface, Lines of curvature. Rodrigues and Monge's theorem, Euler's theorem, Joachimisthal's theorem, Dupin's indicatrix, Third fundamental form.

#### **TEXT BOOKS**

Differential Geometry:
 Differential Geometry of Three Dimensions:
 C.E. Weathrburn

3. Elements of Differential Geometry: R.S. Millman & G.D. Parket

4. Introduction to Differential Geometry: A. Goctz

#### SEMESTER – III

#### PAPER-XVI **FLUID MECHANICS** SOS/ Math/E001

- I. Motion of the cylindrical and Elliptic Cylinders.
- II. Motion of Sphere, Motion of a sphere in an infinite mass of the liquid at rest at infinity. Liquid streaming past a fixed sphere, Equation of motion of a sphere, Pressure distribution.
- III. General theory of stresses and rate of strains: Newton's law of viscosity, state of stress, principal stresses and principal directions, Transformations of two and three stresses components and rate of strain components, Relation between stresses and rate of strain components.
- IV. Navier-Stokes tequations of motion; Energy equation for viscous fluid, Energy dissipation the to viscosity.

## TEXT BOOKS

1. Foundation to Fluid Mechanics: Yuan

2. Text book of Fluid Dynamics: F. Chorltron 3. An Introduction to Fluid Mechanics: G.K. Batchlor

4. Fluid Dynamics: M.D. Raisinghania

#### **SEMESTER-III**

#### PAPER-XVII CALCULUS OF VARIATION

SOS/ Math/E002

- Variation of function: Necessary condition for an extremum. Euler's equation, I. fixed end point problem for unknown functions. Variational problems in parametric form. Functional depending on higher order derivatives and variational problems with subsidiary condition.
- II. The isoperimetric problem, Invariance of Euler's equation under coordinate transformation, General variational of functional, Variable end point problems. Transversality condition transversal theorem, Weierstrass Endmann corner condition.
- III. Cononical form of Euler equations and their first integrals. Cononical transformation, Noether's theorem, The principle of least action, Conservation law, Hamilton Jacobi's equations, Jacobi's theorem.
- IV. The second variation of a functional and the formula for second variation. Legendre's necessary condition. Jacobi's necessary condition, Conjugate point, Sufficient condition for a weak extremum.

# TEXT BOOKS

1. Calculus of Variation: Gelfran and Fomin 2. Calculus of Variation : Esgolac3. Calculus of Variation : Gupta

#### **SEMESTER III**

#### PAPER-XVIII COMPUTER FUNDAMENTALS AND DATA STRUCTURES

SOS/ Math/E003

- I. History and classification of computers, fundamental of computers system: Data types, number system, complements; Floating point representation, normalized floating point representation, fixed point represented arithmetic computations.
- II. Logical gates, Boolean algebra, truth tables, logic diagrams, logical expressions/function, Demorgan's theorem, Karnaugh maps, sum of product and product of sums, combinational circuits, and integrated circuits.
- III. Introduction to data structures, arrays, stack and queues, linked lists, singly and doubly linked lists, binary trees, operations on binary trees and applications.
- IV. Sorting and searching algorithms, and graphs.

#### **TEXT BOOKS**

- 1. Fundamental of Computers, PHT, India: V. Raja Raman.
- 2. Introduction to computers, Mc-Graw-Hill: P. Norton
- 3. Data structures, Algorithms and Application in C++, Univerties Press: S. Sahni
- 4. Data Structures with C (Schaum's series) Tata Mc-Graw-Hill: S. Lipschutz

#### **SEMESTER III**

# PAPER- XIX ALGEBRAIC CODING THEORY

SOS/ Math/E004

- I. The communication channel, The coding problem, Types of Codes, Block Codes, Error-Detecting and Error-Correcting Codes, Linear Codes, The Hamming Metric, Description of Linear Block Codes by Matrices, Dual Codes, Standard Array, Syndrome, Step-by-step Decoding Modular Representation.
- II. Error-Correction Capabilities of Linear Codes, Bounds on Minimum Distance for Block Codes, Plotkin Bound, Hamming sphere packing Bound, Varshamov-Gilbert-Sacks bound, Bounds for Burst-Error Detecting and Correcting Codes, Important Linear Block Codes.
- III. Hamming Codes, Golay Codes, Perfect Codes, Quasi-perfect Codes, Reed-Muller Codes, Codes derived from Hadmard Matrices, Product Codes, Concatenated Codes.
- IV. Tree Codes, Convolutional Codes, Description of Linear Tree and Convolutional Codes by Matrices, Standard Array, Bounds on minimum distance for

Convolutional Codes, V-G-S bound, Bounds for Burst-Error Detecting and Correcting Convolutional Codes.

#### TEXT BOOKS

A First Course in Coding Theory: Raymond Hill
 Error Correcting Coding Theory: Man Young Rhee

3. Error-Correcting Codes: W.W. Peterson and E.J. Weldon, Jr.

4. Algebraic Coding Theory: E.R. Berlekamp

#### **SEMESTER III**

PAPER XX SELF-STUDY(From other Department) Any of the XX(a), XX(b)

& XX(c) Math SOS/

Math/E005

# Paper XX (a) MATHEMATICAL METHOD

- 1. Hermite Polynomial
- 2. Laguerre Polynomial
- 3. Bessel's Polynomial
- 4. Chebyshev Polynomial

# PAPER XX (B) TENSOR ANALYSIS AND SPECIAL THEORY OF RELATIVITY

**SOS./ Math/E005(C)** 

### UNIT I

Invariance – Transformations of coordinates and its properties- Transformation by

invariance – Transformations by covariance and contra variance- Covariance and contra

variance – Tensor and Tensor character of their laws- Algebras of tensors- Quotient

Tensors- Symmetric and skew symmetric tensors- Relative tensors.

#### **UNIT II**

Metric Tensor- The fundamental and associated tensors- Christoffel's symbals-Transformations of Chrisffel's symbols- Covariant Differentiation of Tensors-Formulas for covariant Differentiation- Ricci Theorem- Riemann- Christoffel Tensor and their Properties.

#### **UNIT III**

Einstein Tensor – Riemannian and Euclidean Spaces (Existence Theorem)- The esystems and the generalized Kronecker deltas- Application of the e-systems.

#### **UNIT IV**

Special Theory of Relativity: Galilean Transformation-Maxwell's equations – The ether Theory- The Principle of Relativity Relativistic Kinamatics: Lorentz Transformation equations – Events and simultaneity – Example Einstein Train- Time dilation- Longitudinal Contraction- Invariant Interval- Proper time and Proper distance- World line- Example –twin paradox – addition of velocities- Relativistic Doppler effect.

## **TEXT BOOKS**

- 1. I.S. Sokolnikoff, Tensor Analysis, John Wiley and Sons, New York, 1964
- 2. D. Greenwood, Classical Dynamics, Prentice Hall of India, New Delhi, 1985
- 3. Tensor Calculus, Toronto, 1949 : J.L. Synge and A. Schild
- 4. The Mathematical Theory of Relativity, Cambridge University Press, 1930 : A.S. Eddington.
- 5. An Introduction to Theory of Relativity, New York, 1942: P.G. Bergman
- 6. Riemannian Geometry and Tensor Calculus, Cambridge, 1938: C.E. Weatherburn

# PAPER XX (C) FINANCIAL MATHEMATIS SOS/Math/E005 (c)

#### **UNIT I**

SINGLE PERIOD MODELS: Definitions from Finance- Pricing a forward- one- step Binary Model – a termary Model- Characterization of no arbitrage- Risk-Neutral Probability Measure

#### UNIT II

BINOMIAL TREES AND DISCRETE PARAMETER MARTINGALES: Multiperiod Binary model- American Options- Discrete parameter martingales and Markov processes- Martingale Theorems- Binomial Representation Theorem- Overturn to Continuous models.

# **UNIT III**

BROWNIAN MOTION: Definition of the process- Levy's Construction of Brownian Motion – The Reflection Principle and Scaling – Martingales I Continuous time.

#### **UNIT IV**

STOCHASTIC CALCULUS: Non-differentiability of Stock prices- Stochastic integration- Ito's formula- Integration by parts and stochastic Fubini Theorem-

Girsanov Theorem- Brownian Martingale Representation theorem- Geometric Brownian Motion- The Feynman- Kac Representation.

#### **TEXT BOOKS**

- 1. Alison Etheridge A Course in Financial Calculus, Cambridge University Press, Cambridge, 2002
- 2. Martin Boxter and Andrew Rennie, Financial Calculus: An Introduction to Derivatives Pricing. Cambridge University Press, Cambridge, 1996.
- 3. Damien Lamberton and Bernard Lapeyre, (Translated by Nicolas Rabeau and Farancois Mantion).
- 4. Introduction to Stochastic Calculus Applied to Finance, Chapman and Hall, 1996.
- 5. Marek Musiela and Marek Ruthowshi, Martingale Methods in Financial Madeling, Springer Verlag, New York, 1988.
- 6. Robert J. Elliot and P. Ekkehard Kopp, Mathematics of Financial Markets, Springer, Verlag, New York, 2001 (3<sup>rd</sup> Printing)

PAPER-XXI VIVA-VOCE SOS/Math/C0016

#### SEMESTER -IV

## PAPER-XXII MEASURE AND INTEGRATION-II

SOS/Math/C0017

- I. Convergence in Measure, Egoroff's Theorem, Fatou's Lemma, Convergence Theorems.
- II. Dini derivatives, Differentiation of monotone functions, Functions of bounded variations, Differentiation of an integral, Absolute continuous functions, Integral of the derivative.
- III. L<sup>P</sup>-spaces, Properties of L<sup>P</sup>- spaces, Holder's and Minkowshi's Inequalities.
- IV. Signed measure, Hahn-Decomposition theorem, Radom-Nikodyn theorem, Pruduct measure.

# **TEXT BOOKS**

Real Analysis:
 Measure and Integration:
 S.K. Berberian

3. Lebesgue Measure and Integration : P.K. Jain and V.P. Gupta

4. Measure Theory and Integration : G. De. Barra.

#### **SEMESTER-IV**

# PAPER-XXIII LINEAR INTEGRAL EQUATIONS & BOUNDARY VALUE

PROBLEMS SOS/Math/C0018

- I. Classification of integral equations, Relation between Differential and Integral equations, Green's function.
- II. Solution of Fredholm Integral Equations, Solution of Volterra Integral Equations.
- III. Hilbert-Schmidt Theory and classical theory of Fredholm., Singular Integral equation and Numerical solution of Integral equations.
- IV. Perturbation techniques and its applications to mixed boundary value problems, Two-part and three-part boundary value problems, Solutions o electrostatic problems involving a charged circular disk and annular circular disk, a spherical cap, an annular spherical cap in a free space or a bounded space.

#### TEXT BOOKS

Integral Equations : Hilderbrand
 Linear Integral Equations : V. Lovit
 Linear Integral Equations : R.P. Kanwal
 Integral Equations : Li. G. Chanbers

#### **SEMESTER-IV**

#### PAPER XXIV FUNCTIONAL ANALYSIS SOS/Math/C0019

- I. Normed linear spaces, Banach spaces, Subspaces, Quotient Spaces, Equivalent, Norms.
- II. Bounded linear Transformation/operators, Hahn Banach Theorem, Open mapping, Theorem, Closed Graph Theorem Uniform Boundedness Principle.
- III. Inner Product spaces, Hilbert Spaces, Orthogonality of vectors, orthogonal complements and projection Theorem, Riesz Representation Theorem, Orthogonal Sets.
- IV. Operators on Hilbert Spaces, Self-Adjoint, Normal and unitary operators orthogonal projection operators.

# **TEXT BOOKS**

1. Functional Analysis : P.K. Jain, O.P. Ahuza and Khalil

Ahamad

2. Topology and Modren Analysis : G.F. Simmons

3. Introductory functional Analysis with Applications : E. Kreyszig

4. Functional Analysis: B.V. Limaye

#### **SEMESTER-IV**

#### PAPER XXV BIOMECHANICS SOS/Math/E006

- I. Biomechanics, Method of approach, Tools of investigation, Stresses and rates of strain, Constitutive equations, Newtonian viscous fluid, Hookean eastic solid, Biological transport process, Basic momentum, Heat and mass transport concepts.
- II. Conservation laws: mass conservation, Momentum conservation, Energy conservation.
- III. Biofluid dynamics concept, Transport phenomena and the cardiovascular system.
- IV. Biofluid mechanics of organ systems: The lungs, The Kidneys and the lever.

## **TEXT BOOKS**

1. Biomechanics, Springer-verlag: Y.C. Fung

2. Biofluid Dynamics Taylor and Francis: Clement Kluinstreuer

3. Frontier in Mathematical Biology: S.A. Levin4. BioMathematics: Ricciardi

#### **SEMESTER-IV**

#### PAPER XXVI FUZZY SET THEORY SOS/Math/E007

- I. Fuzzy sets: Basic definitions, a-level sets, convex Fuzzy sets, basic operation on Fuzzy sets, type of Fuzzy sets, Cartesian products, algebraic products, bounded sum and differences, t-norms and t-corners.
- II. The extension principle, The Zadeh's extension principle, Images and inverse image of Fuzzy sets, Fuzzy numbers, element of fuzzy arithmetic.
- III. Fuzzy relation and fuzzy graphs. Fuzzy relation on fuzzy sets, composition of fuzzy relation, min-max composition and properties, equivalence relations, fuzzy compatibility relation.
- IV. Fuzzy logic: An overview of classical logic, multivalued logic, fuzzy propositions, fuzzy qualifiers, linguistic variables and hedge.

#### **TEXT BOOKS**

Fuzzy sets and Fuzzy logic:
 Fuzzy set theory and its Applications:
 Fuzzy set theory, Fuzzy logic and their Applications:
 A.K. Bhargava

#### **SEMESTER-IV**

PAPER XXVII MATHEMATICAL MODELING SOS/Math/E008

**UNIT I** 

Mathematical Modeling through Ordinary Differential Equations of First order: Linear Growth and Decay Models- Non-Linear Growth and Decay Models-Compartment Models- Dynamics problems- Geometrical problems.

#### UNIT II

Mathematical Modeling through Systems of Ordinary Differential Equations of First Order: Population Dynamics- Epidemics- Compartment Models – Economics – Medicine, Arms Race, Battles and International Trade- Dynamics.

#### **UNIT III**

Mathematical Modeling through Ordinary Differential Equations of Second Order: Planetary Motions- Circular Motion and Motion of Satellites- Mathematical Modeling through Linear Differential Equations of Second Order- Miscellaneous Mathematical Models.

#### **UNIT IV**

**Mathematical Modeling through Difference Equations:** Simple Models – Basic theory of Linear Difference Equations with Constant Coefficients- Economics and Finance-Population Dynamics and Genetics- Probability Theory.

#### **TEXT BOOKS**

- 1. J.N. Kapur, Mathematical Modeling, Wiley Eastern Limited, New Delhi, 1988.
- 2. J. N. Kapur, Mathematical Models in Biology and Medicine, Affiliated East-West Press Pvt. Limited, New Delhi, 1981.

#### SEMESTER-IV

#### PAPER XXVIII THEORY OF NUMBERS SOS/MATH/E009

- I. Well ordering Principle, Division Algorithm, Euclidean Algorithm, Fundamental theorem of Arithmetic, Euclid's Lemma,
- II. Congruences, Residue Classes, linear congruences, Euler's Ø function, tau function, Euler's theorem, Fermat's theorem, Wilsons Theorem.

- III. Quadratic congruences beendre symbol, continued fraction, finite and infinite continued fraction.
- IV. Diophantine Equation, Simultaneous linear Diophantine Equations, Sums of Squars, Pell's Equations.

# **TEXT BOOKS**

Elenmentaz number theory : By David M. Burton
 Theory of Numbers : George Andrews
 Elementaz number theory with Application : Thomas Koshy
 Fundamental of Number Theory: William J. Lereque

PAPER XXIX VIVA-VOCE SOS/MATH/C0020

# All India Universities/Colleges of B. Sc and B.A Mathematics unified syllabus will be applied by UGC now.

- Each course will carry 100 marks.
- In each course, sessional test will carry 30 marks, which includes; one
  test of 1 hour duration/assignment / (paper/poster) presentation etc.
  suitable for the course and shall carry 20 marks, 5 marks shall be given
  for participation in academic activities/ discipline and 5 marks for
  attendance in the class.
- In each course, the end semester examination shall be of 70 marks.

Semester	Core Course(04)	Skill Enhancement Course (SEC) (02)	Discipline Specific Elective (DSE) (02)
1	Differential Calculus Credit-06		
2	Differential Equations Credit-06		
3	Real Analysis Credit-06	SEC1 Credit-02	
4	Algebra Credit-06	SEC2 Credit-02	
5		SEC3 Credit-02	DSE1A Credit-06
6		SEC4 Credit-02	DSE1B Credit-06

# **Discipline Specific Electives (DSE)**

# DSE 1A (choose one)

- 1. Matrices
- 2. Mechanics
- 3. Linear Algebra

# DSE 1B (choose one)

- 1. Numerical Methods
- 2. Complex Analysis
- 3. Linear Programming

# **Skill Enhancement Course (SEC)**

# SEC 1 (choose one)

- 1. Logic and Sets
- 2. Analytical Geometry
- 3. Integral Calculus

# SEC 2 (choose one)

- 1. Vector Calculus
- 2. Theory of Equations
- 3. Number Theory

# SEC 3 (choose one)

- 1. Probability and Statistics
- 2. Mathematical Finance
- 3. Mathematical Modeling

# SEC 4 (choose one)

- 1. Boolean Algebra
- 2. Transportation and Game Theory
- 3. Graph Theory

#### **Core 1.1: Differential Calculus**

Limit and Continuity ( $\varepsilon$  and  $\delta$  definition), Types of discontinuities, Differentiability of functions, Successive differentiation, Leibnitz's theorem, and its applications to problems of type  $e^{ax+b}\sin x$ ,  $e^{ax+b}\cos x$ ,  $(ax+b)^n\cos x$ .

Partial differentiation, Euler's theorem on homogeneous functions.

Tangents and normals, Curvature, Asymptotes, Singular points, Tracing of curves. Parametric representation of curves and tracing of parametric curves, Polar coordinates and tracing of curves in polar coordinates. Rolle's theorem, Mean Value theorems, Taylor's theorem with Lagrange's and Cauchy's forms of remainder, Taylor's series, Maclaurin's series of sin x, cos x,  $e^{ax}$ ,  $\log(1+x)$ ,  $(1+x)^m$ .

Maxima and Minima, Indeterminate forms concavity and inflection points, asymptotes, curve tracing in Cartesian coordinates, tracing in polar coordinates of standard curves, L'Hospital's rule and applications in business, economics and life sciences.

# **Core 2.1: Differential Equations**

First order exact differential equations. Integrating factors, rules to find an integrating factor. First order higher degree equations solvable for x, y, p. Methods for solving higher-order differential equations. Basic theory of linear differential equations, Wronskian, and its properties. Solving a differential equation by reducing its order. Linear homogenous differential equations with constant coefficients, Linear non-homogenous differential equations.

The method of variation of parameters, The Cauchy-Euler equation, Simultaneous differential equations, Total differential equations.

Order and degree of partial differential equations, Concept of linear and non-linear partial differential equations, Formation of first order partial differential equations, Linear partial differential equation of first order, Lagrange's method, Charpit's method. Equilibrium points, Interpretation of the phase plane, predatory-prey model and its analysis, epidemic model of influenza and its analysis, battle model and its analysis

# **Core 3.1:** Real Analysis

Finite and infinite sets, examples of countable and uncountable sets. Real line, bounded sets, suprema and infima, completeness property of R, Archimedean

property of R, intervals. Concept of cluster points and statement of Bolzano-Weierstrass theorem.

Real Sequence, Bounded sequence, Cauchy convergence criterion for sequences. Cauchy's theorem on limits, order preservation and squeeze theorem, monotone sequences and their convergence (monotone convergence theorem without proof).

**Infinite series** Cauchy convergence criterion for series, positive term series, geometric series, comparison test, convergence of p-series, Root test, Ratio test, alternating series, Leibnitz's test (Tests of Convergence without proof). Definition and examples of absolute and conditional convergence. Sequences and series of functions, Pointwise and uniform convergence. Mn-test and M-test.

# Core 4.1: Algebra

Definition and examples of groups, examples of abelian and non-abelian groups, the group Zn of integers under addition modulo n and the group U(n) of units under multiplication modulo n. Cyclic groups from number systems, complex roots of unity, circle group, the general linear group GLn (n,R), groups of symmetries of (i) an isosceles triangle, (ii) an equilateral triangle, (iii) a rectangle, and (iv) a square, the permutation group Sym (n), Group of quaternions.

Subgroups, cyclic subgroups, the concept of a subgroup generated by a subset and the commutator subgroup of group, examples of subgroups including the center of a group. Cosets, Index of subgroup, Lagrange's theorem, order of an element, Normal subgroups: their definition, examples, and characterizations, Quotient groups.

Definition and examples of rings, examples of commutative and non-commutative rings: rings from number systems, Zn the ring of integers modulo n, ring of real quaternions, rings of

matrices, polynomial rings, and rings of continuous functions. Subrings and ideals, Integral domains and fields, examples of fields: Zp, Q, R, and C.

#### **DSE 1A.1: Matrices**

R, R<sup>2</sup>, R<sup>3</sup> as vector spaces over R. Standard basis for each of them. Concept of Linear Independence and examples of different bases. Subspaces of R<sup>2</sup>, R<sup>3</sup>.

Translation, Dilation, Rotation, Reflection in a point, line and plane. Matrix form of basic geometric transformations. Interpretation of eigen values and eigen vectors for such transformations and eigen spaces as invariant subspaces. Types of matrices. Rank of a matrix. Invariance of rank under elementary transformations. Reduction to normal form, Solutions of a system of linear equations using matrices. Solutions of linear homogeneous and non-homogeneous equations with number of equations and unknowns upto four. Matrices in diagonal form. Reduction to diagonal form upto matrices of order 3. Computation of matrix inverses using elementary row operations.

#### **DSE 1A.2: Mechanics**

Conditions of equilibrium of a particle and of coplanar forces acting on a rigid Body, Laws of friction, Problems of equilibrium under forces including friction, Centre of gravity, Work and potential energy. Velocity and acceleration of a particle along a curve: radial and transverse components (plane curve), tangential and normal components (space curve), Simple harmonic motion, Simple Pendulum.

# **DSE 1A.3: Linear Algebra**

Vector spaces, subspaces, algebra of subspaces, quotient spaces, linear combination of vectors, linear span, linear independence, basis and dimension, dimension of subspaces. Linear transformations, null space, range, rank and nullity of a linear transformation, matrix representation of a linear transformation, algebra of linear transformations. Dual Space, Dual Basis, Double Dual, Eigen values and Eigen vectors, Characteristic Polynomial. Isomorphisms, Isomorphism theorems, invertibility and isomorphisms, change of coordinate matrix.

#### **DSE 1B.1: Numerical Methods**

Algorithms, Convergence, Bisection method, False position method, Fixed point iteration method, Newton's method, Secant method, LU decomposition, Gauss-Jacobi, Gauss-Siedel and SOR iterative methods.

Lagrange and Newton interpolation: linear and higher order, finite difference operators.

Numerical differentiation: forward difference, backward difference and central Difference.

Integration: trapezoidal rule, Simpson's rule, Euler's method.

# **DSE 1B.2: Complex Analysis**

Limits, Limits involving the point at infinity, continuity. Properties of complex numbers, regions in the complex plane, functions of complex variable, mappings. Derivatives, differentiation formulas, Cauchy-Riemann equations, sufficient conditions for differentiability.

Analytic functions, examples of analytic functions, exponential function, Logarithmic function, trigonometric function, derivatives of functions, definite integrals of functions. Contours, Contour integrals and its examples, upper bounds for moduli of contour integrals. Cauchy- Goursat theorem, Cauchy integral formula. Liouville's theorem and Taylor and Laurent series, and its examples.

# **DSE 1B.3: Linear Programming**

Linear Programming Problems, Graphical Approach for Solving some Linear Programs. Convex Sets, Supporting and Separating Hyperplanes. Theory of simplex method, optimality and unboundedness, the simplex algorithm, simplex method in tableau format, introduction to artificial variables, two-phase method, Big-M method and their comparison. Duality, formulation of the dual problem, primal-dual relationships, economic interpretation of the dual.

## **SEC 1.1: Logic and Sets**

Introduction, propositions, truth table, negation, conjunction and disjunction. Implications, biconditional propositions, converse, contra positive and inverse propositions and precedence oflogical operators. Propositional equivalence: Logical equivalences. Predicates and quantifiers: Introduction, Quantifiers, Binding variables and Negations.

Sets, subsets, set operations, the laws of set theory and Venn diagrams. Examples of finite and infinite sets. Finite sets and counting principle. Empty set, properties of empty set. Standard set operations. Classes of sets. Power set of a set. Difference and Symmetric difference of two sets. Set identities, Generalized union and intersections. Relation: Product set, Composition of relations, Types of relations, Partitions, Equivalence Relations with example of congruence modulo relation.

# **SEC 1.2: Analytical Geometry**

Techniques for sketching parabola, ellipse and hyperbola. Sphere, Cone, Cylindrical Surfaces, Central Conicoids.

# **SEC 1.3: Integral Calculus**

Integration by Partial fractions, integration of rational and irrational functions. Properties of definite integrals. Reduction formulae for integrals of rational, trigonometric, exponential and logarithmic functions and of their combinations. Areas and lengths of curves in the plane, volumes and surfaces of solids of revolution. Double and Triple integrals.

#### **SEC 2.1: Vector Calculus**

Differentiation and partial differentiation of a vector function. Derivative of sum, dot product and cross product of two vectors. Gradient, divergence and curl.

# **SEC 2.2: Theory of Equations**

General properties of polynomials, Graphical representation of a polynomials, maximum and minimum values of a polynomials, General properties of equations, Descarte's rule of signs positive and negative rule, Relation between the roots and the coefficients of equations.

Symmetric functions, Applications symmetric function of the roots, Transformation of equations. Solutions of reciprocal and binomial equations. Algebraic solutions of the cubic and biquadratic. Properties of the derived functions.

# **SEC 2.3: Number Theory**

Division algorithm, Lame's theorem, linear Diophantine equation, fundamental theorem of arithmetic, prime counting function, statement of prime number theorem, Goldbach conjecture, binary and decimal representation of integers, linear congruences, complete set of residues.

Number theoretic functions, sum and number of divisors, totally multiplicative functions, definition and properties of the Dirichlet product, the Möbius inversion formula, the greatest integer function, Euler's phi-function.

# **SEC 3.1: Probability and Statistics**

Sample space, probability axioms, real random variables (discrete and continuous), cumulative distribution function, probability mass/density functions, mathematical expectation, moments, moment generating function, characteristic function, discrete distributions: uniform, binomial, Poisson, continuous distributions: uniform, normal, exponential. Joint cumulative distribution function and its properties, joint probability density functions, marginal and conditional distributions, expectation of function of two random variables, conditional expectations, independent random variables.

#### **SEC 3.2: Mathematical Finance**

Basic principles: Comparison, arbitrage and risk aversion, Interest (simple and compound, discrete and continuous), time value of money, inflation, net present value, internal rate of return (calculation by bisection and Newton-Raphson methods), comparison of NPV and IRR. Bonds, bond prices and yields. Floating-rate bonds, immunization. Asset return, short selling, portfolio return, (brief introduction to expectation, variance, covariance and correlation), random returns, portfolio mean return and variance, diversification, portfolio diagram, feasible set.

# **SEC 3.3: Mathematical Modeling**

Applications of differential equations: the vibrations of a mass on a spring, mixture problem, free damped motion, forced motion, resonance phenomena, electric circuit problem, mechanics of simultaneous differential equations. Applications to Traffic Flow. Vibrating string, vibrating membrane, conduction of heat in solids, gravitational potential, conservation laws.

# SEC 4.1: Boolean Algebra

Definition, examples and basic properties of ordered sets, maps between ordered sets, duality principle, maximal and minimal elements, lattices as ordered sets, complete lattices, lattices as algebraic structures, sublattices, products and homomorphisms. Definition, examples and properties of modular and distributive lattices, Boolean algebras, Boolean polynomials, minimal forms of Boolean

polynomials, Quinn-McCluskey method, Karnaugh diagrams, switching circuits and applications of switching circuits.

# **SEC 4.2: Transportation and Game Theory**

Transportation problem and its mathematical formulation, northwest-corner method, least cost method and Vogel approximation method for determination of starting basic solution, algorithmfor solving transportation problem, assignment problem and its mathematical formulation, Hungarian method for solving assignment problem.

Game theory: formulation of two person zero sum games, solving two person zero sum games, games with mixed strategies, graphical solution procedure.

# **SEC4.3:** Graph Theory

Definition, examples and basic properties of graphs, pseudographs, complete graphs, bi-partite graphs, isomorphism of graphs, paths and circuits, Eulerian circuits, Hamiltonian cycles, the adjacency matrix, weighted graph, travelling salesman's problem, shortest path, Dijkstra's algorithm, Floyd-Warshall algorithm.

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